

# Econ 330: Urban Economics

## Lecture 05

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21 October, 2021

# Lecture 05: Rents

# Schedule

## Today:

(i). Intro to rents

(ii). City "shape"

## Upcoming:

- **Reading** (Chapter 4)
- **PS01 due tonight at 12pm**
- **Be on the look out for PS02**

# Introduction: City shape

**First Week:** philosophical questions

- What is a city?
- Why do cities exist?
- What determines city size?
- How do cities grow?

**Moving forward:**

- What economic forces determine **city shape**?
  - Why does the price of land change?
  - Why are buildings taller in city centers?

**Questions?**

# Introduction: City shape

**Why do people and firms choose a particular location?**

**What influences these choices?**

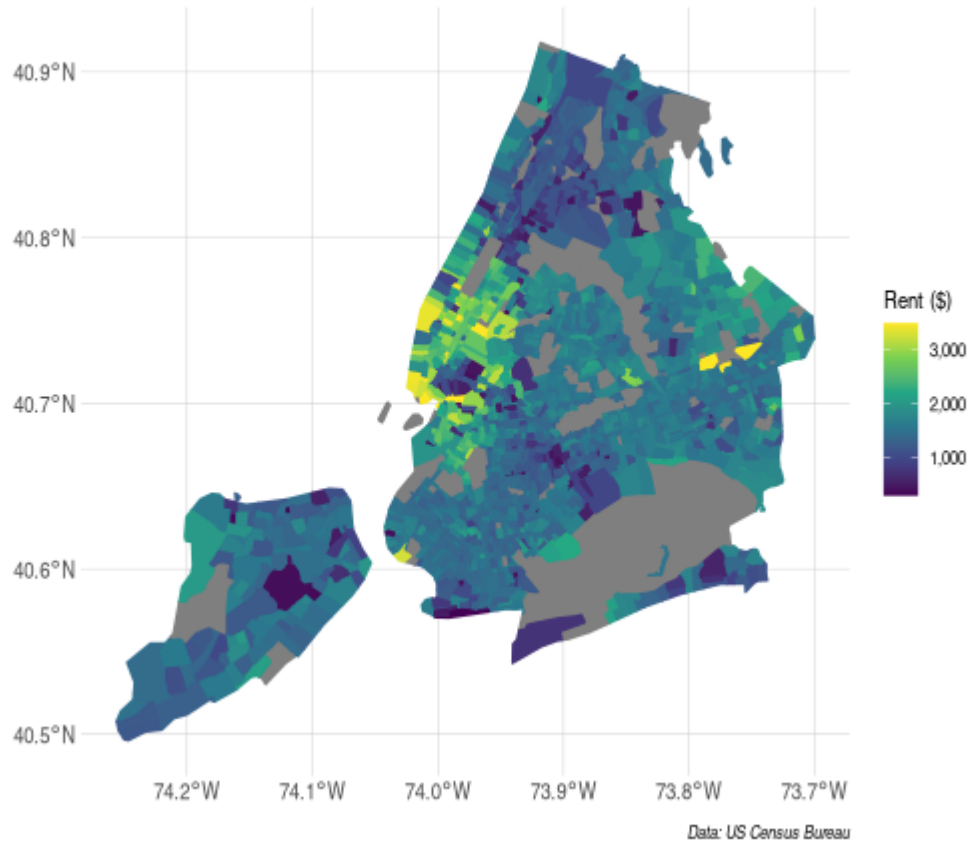
**Can we explain the current *and* historical "shape" of cities?**

- **Today:** How do these choices impact rental prices **within** cities
- **Later:** How do these choices impact rental prices **across** cities
  - Basic introduction into **discrete choice theory**\*

# Data

# Data: NYC

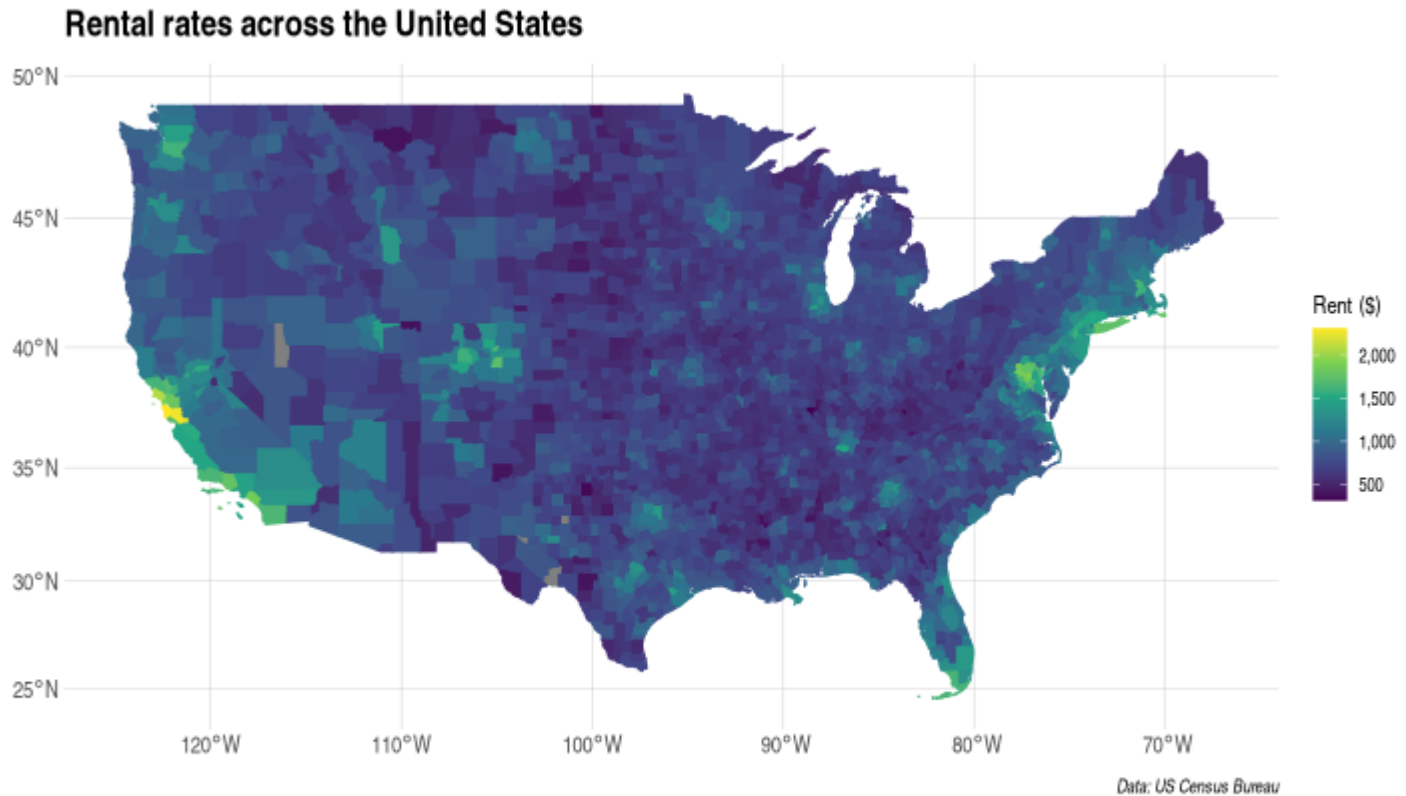
Rental rates across NYC



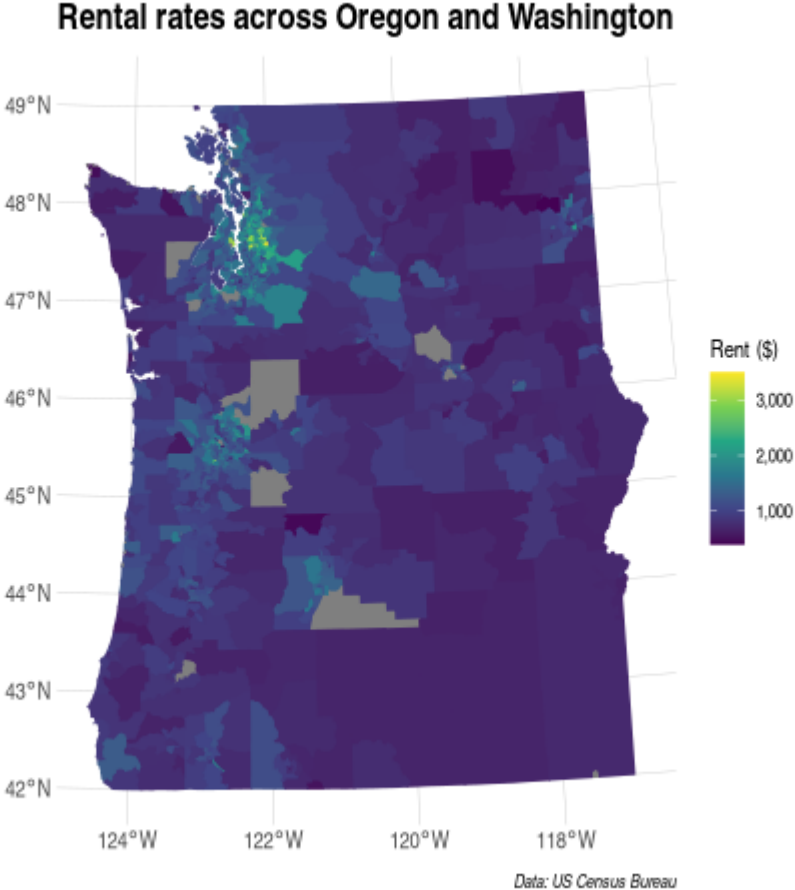
# Data: NYC



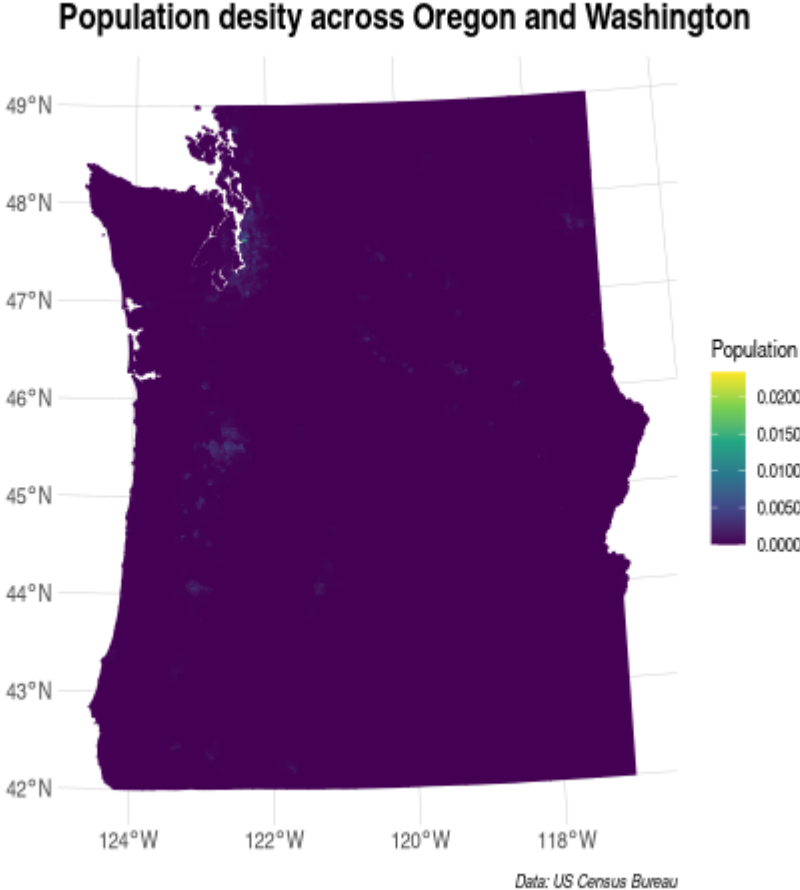
# Data: US



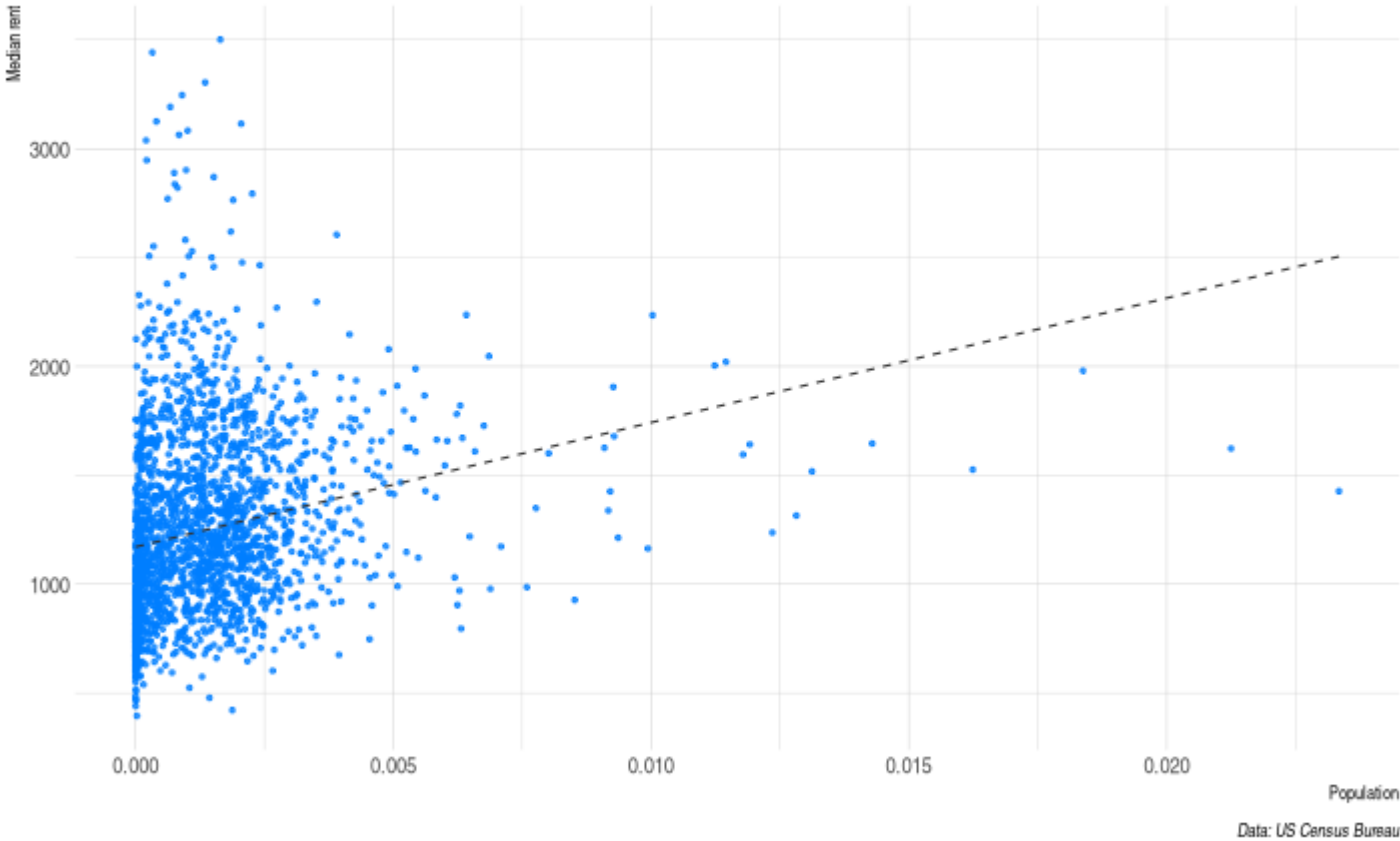
# Data: OR, WA



# Data: OR, WA



### Correlation between rent and population density in OR and WA



# Leaflet

Data

# Bid-Rent Curves

# The Bid-Rent Curve

A **Bid - Rent Curve**: The relationship between rental prices and the distance of land from the city center †

## These curves vary across sectors:

- **Housing**: Accessibility to employment (low commuting costs)
- **Industrial Space**: Accessibility to consumers and suppliers
- **Tech/Office Space**: Accessibility to information

## But first a super simple agricultural land rent model

† It actually does not have to be the city center -- can be a point of attraction. In this class we will always use the city center though.

# Agricultural land rent model

## Definitions:

- **Land rent:** Periodic payment by a land consumer to a landowner
- **Market value:** The amount paid to become the landowner

## Setup:

Rent on a plot of land is determined by how productive the plot is

- **Agriculture:** Price of plot is determined by fertility

**Consider a setting** where farmers grow corn on two types of land

- High fertility (HF): Produces 4 units of corn
- Low fertility (LF): Produces 2 units of corn



# Agricultural land rent model

## Assumptions:

- **(i).** Farmers rent from landowners  $TC_{-r} = 15$  (excluding rent)
- **(ii).** No barriers to the corn market
- **(iii).** Perfect competition  $TC = 15 + r$  &  $P_{corn} = 10$

## How much will farmers bid for land?

**Revenue:**  $TR = P_{corn} \cdot Q_{corn}$

- HF:  $TR_{HF} = 10 \cdot 4 = 40$
- LF:  $TR_{LF} = 10 \cdot 2 = 20$

**Profit:**  $\Pi = TR - TC$

- $\Pi_{HF} = TR_{HF} - TC = 40 - 15 - r$
- $\Pi_{LF} = TR_{LF} - TC = 20 - 15 - r$

**Recall A05: Competition drives economic profit to zero**

# Agricultural land rent model

The following table computes maximum WTP for rent:

**TABLE 6-1** Fertility and Land Rent

	Price of Corn	Quantity Produced	Total Revenue	Nonland Cost	WTP for Land	Bid Rent for Land
Low fertility	\$10	2	\$20	\$15	\$ 5	\$ 5
High fertility	\$10	4	\$40	\$15	\$25	\$25

Since there are no barriers to entry, more firms will enter

- $\Pi \rightarrow 0$
- $\Pi_{HF} = TR_{HF} - TC = 40 - 15 - r = 0 \Rightarrow r = 15$
- $\Pi_{LF} = TR_{LF} - TC = 20 - 15 - r = 0 \Rightarrow r = 5$

# (i) Housing prices model

Extend the bid-rent model to the housing sector within a city

In cities WTP for land depends on **accessibility** rather than productivity

## **Assumptions:**

**(i).** Commuting costs are the **only** location factor in decision making

**(ii).** Only one member of household commutes to employment area

**(iii).** They only consider the monetary cost of commuting (no time cost)

**(iv).** Noncommuting travel is insignificant

**(v)** Public services, taxes, amenities are the same everywhere

**Assumptions ensure the employment area is the focal point of the city**

# (i) Housing prices model: Indifference

**A1:** *Housing prices adjust until there is locational indifference*

- Locational Eq
- IE: A marginal increase in rent just offsets the lower commuting costs

**We call this the locational equilibrium condition.** In math:

$$\Delta P \cdot h + \Delta x \cdot t = 0$$

- $P$ : **Price** of housing (per  $ft^2$ )
- $x$ : **Distance** of commute (miles)
- $h$ : **Housing quantity** (  $ft^2$  )
- $t$ : **Commuting costs** (per mile)

# (i) Housing prices model: Bid-Rent

With locational indifference, we can derive the **slope** of the **bid-rent** curve:

$$\underbrace{\Delta P \cdot h}_{\text{Marginal change in housing cost}} + \underbrace{\Delta x \cdot t}_{\text{Marginal change in commuting cost}} = 0$$

# (i) Housing prices model: Bid-Rent

With locational indifference, we can derive the **slope** of the **bid-rent** curve:

$$\begin{aligned}\Delta P \cdot h + \Delta x \cdot t &= 0 \\ \Delta P \cdot h &= -\Delta x \cdot t\end{aligned}$$

# (i) Housing prices model: Bid-Rent

With locational indifference, we can derive the **slope** of the **bid-rent** curve:

$$\begin{aligned}\Delta P \cdot h + \Delta x \cdot t &= 0 \\ \Delta P \cdot h &= -\Delta x \cdot t \\ \frac{\Delta P}{\Delta x} &= -\frac{t}{h}\end{aligned}$$

**Notice:**  $\frac{\Delta P}{\Delta x}$  is the **slope** of the **bid-rent** curve

**Note:** Price on the vertical axis, distance on the horizontal. Rise over run

$\Delta P \cdot h = -\Delta x \cdot t$ : Another way of putting this:  $MC = MB$ !

# (i) Housing prices model: Bid-Rent

**Alternatively:** Suppose you have decided that the optimal amount of money to spend on housing and commuting per month is  $M^*$

- You can allocate this as

$$P \cdot h + x \cdot t = M^*$$

- Since we graph the bid rent curve in the  $(x, P)$  space, we solve for  $P$ :

$$P \cdot h + x \cdot t = M^*$$

$$P \cdot h = M^* - x \cdot t$$



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$$P \cdot h = M^* - x \cdot t$$

$$P = \frac{M^*}{h} - \frac{t}{h} \cdot x$$

- Slope:  $\Delta P = 0 - \frac{t}{h} \cdot \Delta x \implies \frac{\Delta P}{\Delta x} = -\frac{t}{h}$

We can use calculus and take derivative of  $P$  w.r.t  $x$  and get the same thing

# (i) Housing prices model: Example

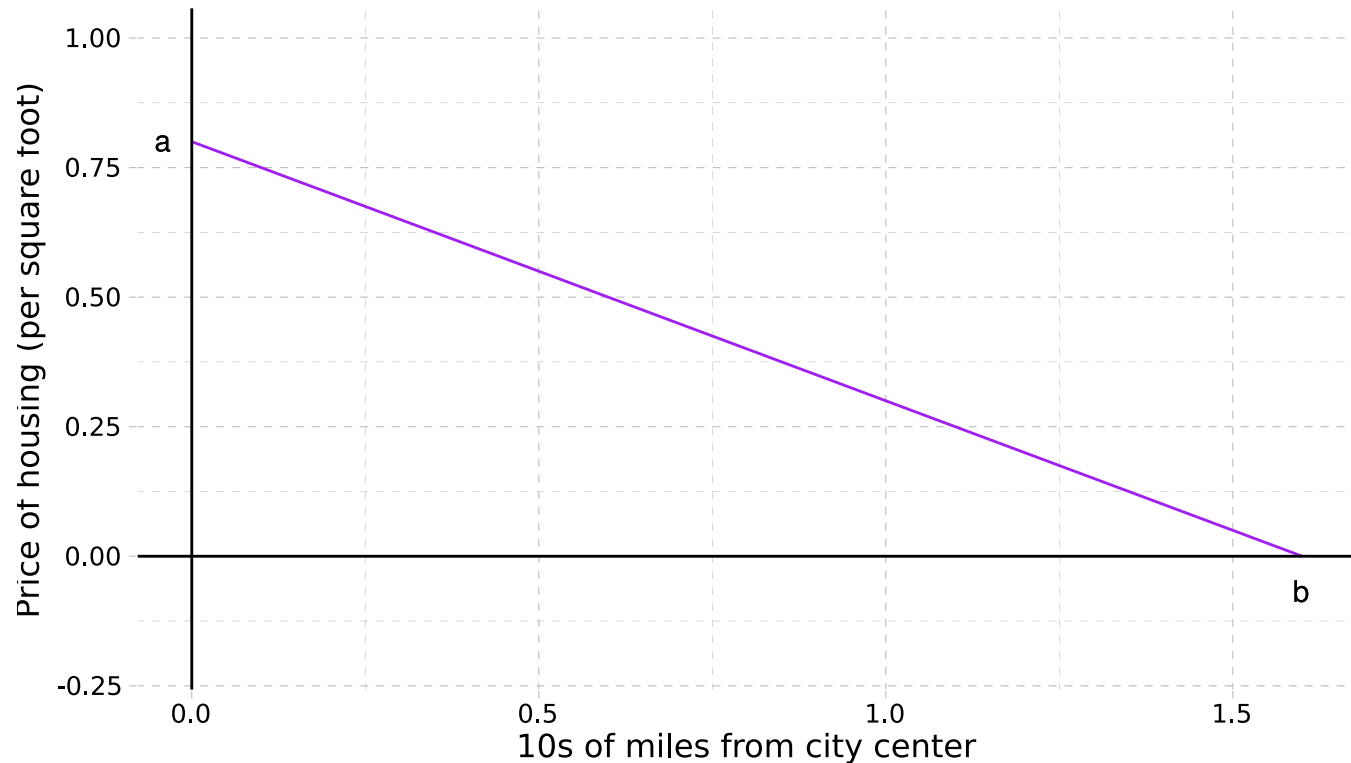
## Suppose the following:

- Each household has \$800 a month to spend
  - Split between housing and commuting
- All rental units are the same size (*1000 sq/ft*), one HH occupying each
- Monthly commuting cost is \$50 dollars per mile from city center

**Task:** Draw the housing - price curve.

- Put miles from city center on **x axis** and price per square foot on **y axis**

# Example: The housing price curve



**a:** Max WTP for a square foot (at center of city) (80 c per square foot)

**b:** Furthest away from center HH is willing to live (16 miles)

# (i) Housing prices model: Substitution

Q: If you are trying to move to a better area, would you choose to sacrifice ("*substitute*") apartment size?

A: Probably yes. Most people are willing to substitute

Q: What do I mean by substitute? Substitute what?

A: Substitute housing consumption for lower commuting cost

( or anything else being close to the center of the city gets you)

# (i) Housing prices model: Substitution

Let's formalize the mechanism for substitution a bit:

**higher prices**  $\implies$  **higher opportunity cost** per square foot of housing

- As rent  $\uparrow$ , consumers are likely to substitute towards other goods
  - decreasing the square footage of housing demanded

**Housing units closer to city centers are thus likely to be smaller in size**

# Adding substitution to the model

Q: Did our model of locational indifference accommodate for substitution?

$$\Delta P \cdot h + \Delta x \cdot t = 0$$

A: No because  $h$  (*quantity of housing consumed*) is **independent of distance** from center,  $x$

- $h$  is exogenous in the model

*If consumers can substitute*, our locational indifference condition becomes:

$$\Delta P \cdot h(x) + \Delta x \cdot t = 0$$

Where  $h(x)$  is an *increasing* function of  $x$

**Ex:**  $h(10) > h(5)$

- Quantity of housing demanded 10 miles away exceeds that of 5 miles

# Manufacturing Bid-Rent

# Manufacturing Bid Rent

WTP for land from manufacturing firms is a function of accessibility

**Fact:** Urban manufacturing employment is largely decentralized

Most firms locate close to the highway. **Why?**

Firms are balancing **freight** and **labor costs**

- Further from labor  $\implies$  **higher wage** (compensating commuting costs)
- Further from shipping center  $\implies$  higher freight cost



# Manufacturing Bid Rent

Let's start with a simple model. **Assumptions:**

**(i). Input prices, output prices & quantities** are fixed

- Firms only decides location

**(ii).** Firms import and export output to/from cities via a **central terminal**

- Train, harbor, etc.
- Horse drawn carts are used to bring goods to terminal

**(iii).** Wage are a function of commute time.

- Wage is highest at center

# Firm's Bid Rent

What do we use to get the firm's bid - rent equation?

## **A5: Competition generates zero economic profit**

Recall the profit equation:

$$\pi = TR - TC$$

In this model:

- $TR = P * Q$  (fixed, exogenous, i will tell you)
- $TC$  is a function of freight cost, labor cost, and intermediate goods cost

$$TC(x) = \text{Freight Cost}(x) + \text{Labor Cost}(x) + \text{Land Cost}(x) + \text{Intermediate Input Cost}$$

# Firm Bid Rent

From here on out, let's call **Intermediate Input Cost** =  $\bar{I}$

- Invoking zero economic profit, from the last slide we can write:

$$TR - (\text{Freight Cost}(x) + \text{Labor Cost}(x) + \text{Land Cost}(x) + \bar{I}) = 0$$

**In words:** The most a firm would be willing to pay for land then is revenue net of non land cost

Rearranging:

$$\text{Land Cost}(x) = TR - \text{Freight Cost}(x) - \text{Labor Cost}(x) - \bar{I}$$

**Note:** Land Cost =  $P(x) * L_m$ , where:

- $P(x)$  is the *price of land at x miles away from the center*
- $L_m$  is the *amount of land the manufacturer uses in production*

# Firm Bid Rent: Equation

We can replace land cost with  $P(x) * L_m$  to get the equation for the **manufacturing bid rent** curve

$$P(x) * L_m = TR - \text{Freight Cost}(x) - \text{Labor Cost}(x) - \bar{I}$$

# Firm Bid Rent: Equation

We can replace land cost with  $P(x) * L_m$  to get the equation for the **manufacturing bid rent** curve

$$P(x) * L_m = TR - \text{Freight Cost}(x) - \text{Labor Cost}(x) - \bar{I}$$
$$P(x) = \frac{TR - \text{Freight Cost}(x) - \text{Labor Cost}(x) - \bar{I}}{L_m}$$

## Comparative statics:

**In words**, this equation says:

- Higher revenues  $\implies$  higher land prices **for every distance**  $x$
- An increase in freight costs, labor costs, or intermediate input costs will **decrease** the price for every distance  $x$

# Example

## Suppose:

$$P = 5, Q = 2, FC(x) = 4x, \text{Labor}(x) = 1 - 3x, L_m = 1, \bar{I} = 0$$

- (i). Derive the firm's bid rent curve. Carefully write down your steps
- (ii). What is the price the firm is willing to pay for land at  $x = 1$ ?
- (iii). Is the WTP higher or lower when we move away from the center?
- (iv). What distance away from the center is the WTP zero?

# Example

**(i).** Start with zero profit condition:

$$\pi(x) = 0 \implies TR - FC(x) - LC(x) - P(x) = 0$$

Plugging in:

$$\begin{aligned} 5 \times 2 - 4x - (1 - 3x) - P(x) &= 0 \\ 9 - x &= P(x) \end{aligned}$$

**(ii).**  $P(1) = 8$

**(iii).** Lower (if  $x_2 > x_1$ ,  $P(x_2) < P(x_1)$ )

**(iv).**  $P(x) = 0 \implies x = 9$

# Back to Reality

How can a model like this help us understand the industrial revolution?

- What happened to freight costs? **They fell** A few innovations:

## **Transportation Innovations:**

- Omnibus (1827)
- Cable Cars (1873)
- Electric Trolley (1886)
- Subways (1895)

In our model, what do these innovations do?

**Decrease labor costs relative to freight**



# More History

The *intracity* truck (1910): 2x faster and half as costly as the horse-drawn wagon<sup>†</sup>

- Decreased cost of moving output **relative** to cost of moving workers
- Manufacturing firms moved closer to low-wage suburbs

*Intercity* truck (1930): alternative to ships and rail<sup>††</sup>

- **Highways**: Industry shifted from **ports** and **railroad terminals** to **roads**
- **Modern cities**: Industry oriented toward highways and beltways
  - Freight costs decreased relative to labor

<sup>†</sup> Intra = Within <sup>††</sup> Inter = Across

(iii) Office space bid-rent

# (iii) Office space bid-rent

Final rent bidders we will consider - **offices**

Same as the other bidders, WTP for land depends on accessibility

## **Why?**

Office firms use high skilled labor- *face to face* interaction is productive

- Proximity to other office firms is an important input

**Oppurtunity cost** of high skilled labor is greater than other types of labor

# Office Bid Rent

So as office firms get further from center their "transit" cost goes up. So what must happen to WTP?

# City Organization

**How do we put all of this together?**

**Why are these called bid rent curves?**

**Land will be allocated to highest bidder**

- This will vary by location in the city

**Example:** Assume profit for office and manufacturing is given by

$$\pi_{\text{office}} = 105 - P(x_{\text{Office}}) - (5 + 4 \times x_{\text{office}})$$

$$\pi_{\text{manufact}} = 75 - P(x_{\text{manufact}}) - (5 + 2 \times x_{\text{manufact}})$$

For consumers, they can allocate money between housing and commuting:

$$r(x_{\text{commuter}}) = \frac{50}{2} - \frac{1}{2} \times x_{\text{commuter}}$$

# Example

$$\pi_{\text{office}} = 105 - r(x_{\text{Office}}) - (5 + 4 \times x_{\text{office}})$$

$$\pi_{\text{manufact}} = 75 - r(x_{\text{manufact}}) - (5 + 2 \times x_{\text{manufact}})$$

$$r(x_{\text{commuter}}) = \frac{100}{4} - \frac{2}{4} \times x_{\text{commuter}}$$

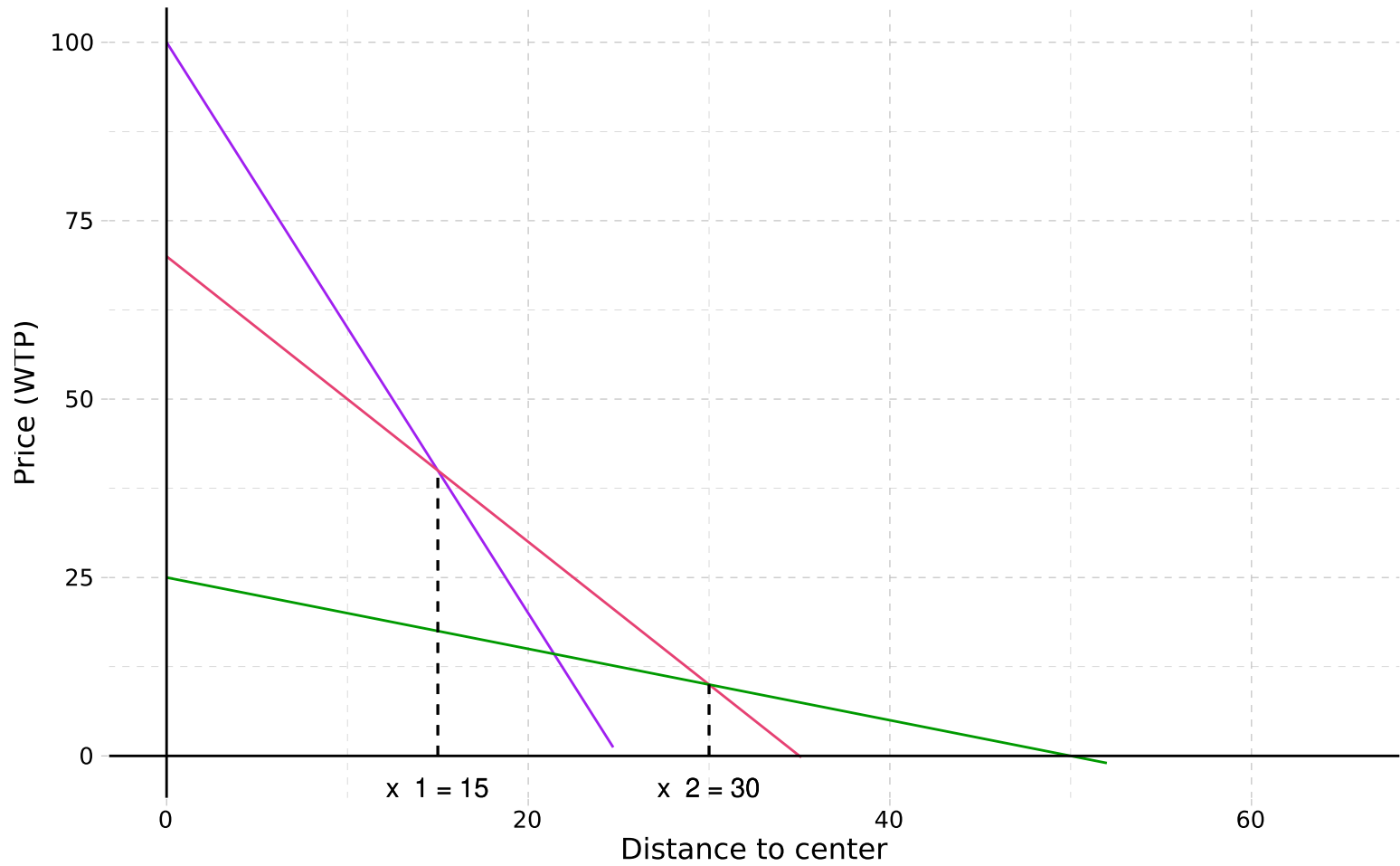
## Task:

**(i).** Derive the bid rent curve for office space, manufacturing, and commuters. Plot all of them.

**(ii).** Find how land is allocated. What range from the center is:

- Office space
- Manufacturing space
- Housing space?

# Example



# Example

Bid rent curves for office and manufacturing come from zero profit. Commuters curve was given.

- Office:  $r(x_{\text{office}}) = 105 - (5 + 4 \times x_{\text{office}})$
- Manufacturing:  $r(x_{\text{manufact}}) = 75 - (5 + 2 \times x_{\text{manufact}})$
- Commuters: (given)  $r(x_{\text{commuter}}) = \frac{100}{4} - \frac{2}{4} \times x_{\text{commuter}}$
- Office firms locate in the range of  $x$  in  $[0, 15]$
- Manufacturing firms locate in the range of  $x$  in  $[15, 30]$
- Commuters locate in the range of  $x$  in  $[30, 50]$



# Bonus: COVID19 and Cities research

## Questions:

**Q1)** How does COVID19 impact housing/rental prices?

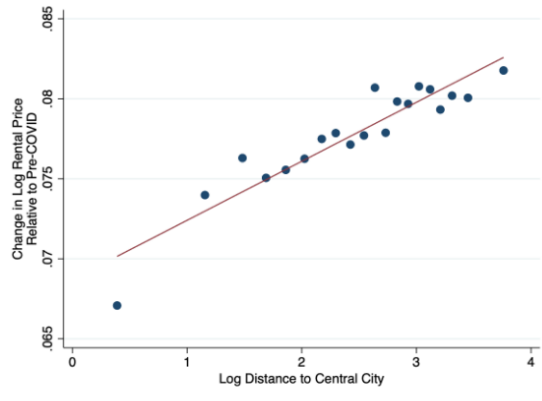
- Is the effect the same everywhere? Why or why not?

**Q2)** How many jobs can be done remotely? Does this vary systematically across sectors? Cities?

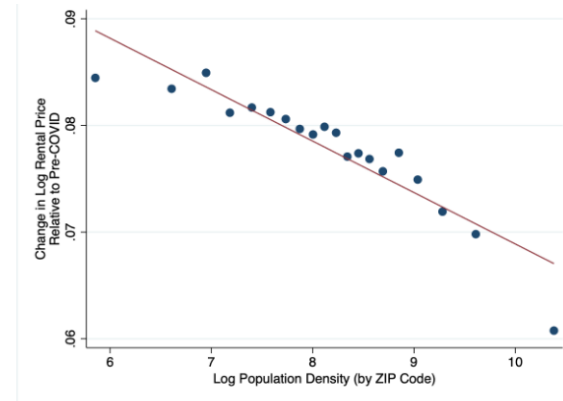
**Q3)** What do we think will happen to city structure as a result of increased (potentially permanent) WFH

# Bonus: COVID19 and Cities Research

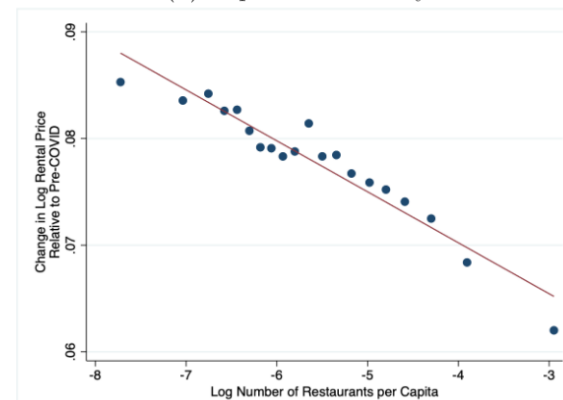
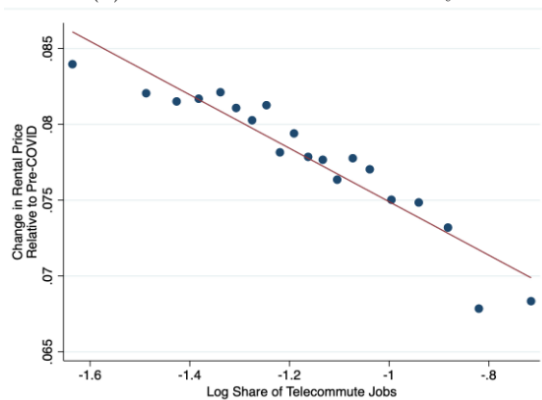
- A1:** *The Impact of the COVID-19 Pandemic on the Demand for Density: Evidence from the U.S. Housing Market (Liu & Su, 2020)*



(a) Distance to the Central City

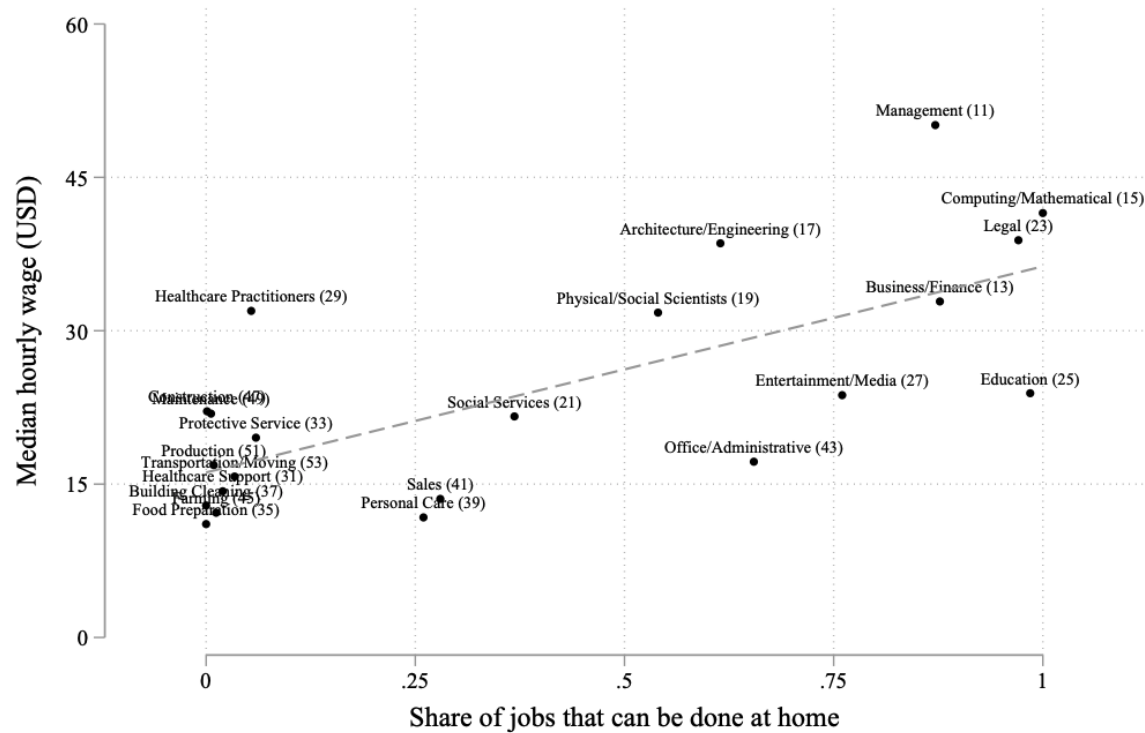


(b) Population Density



# Bonus: COVID19 and Cities research

- **A2:** *How many jobs can be done at home?* (Dingel & Nieman, 2020)

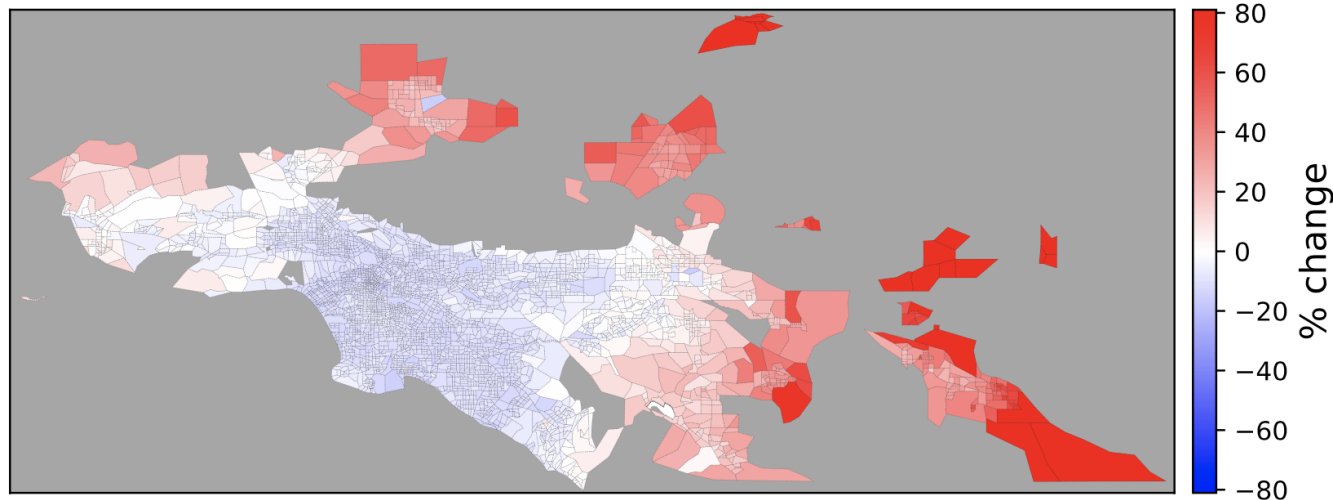


# Bonus: COVID19 and Cities research

**Q3)** What do we think will happen to city structure as a result of increased (potentially permanent) WFH?

- **A3:** *How Do Cities Change When We Work from Home?* (Delventhal et. al, 2020)

Figure 2: House prices



*Note:* Percentage change relative to benchmark economy in counterfactual with  $\psi = 0.33$ . See main text for details.

# Checklist

- 1)
- 1.5)
- 2)
- 3)
- 4)